6.4: WAVEGUIDE PERTURBATION TECHNIQUES IN MICROWAVE SEMI-CONDUCTOR DIAGNOSTICS*

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DC transport properties (e.g., conductivity, Hall effect, magneto-conductivity) are proportional to various averages of the electron-lattice relaxation time ($\langle\tau\rangle$, $\langle\tau^2\rangle$, $\langle\tau^3\rangle$, etc.) and hence give indirect information about the scattering mechanisms affecting the conduction process. With microwaves, the observation frequency can frequently be of the order of the scattering frequency $1/(2\pi\langle\tau\rangle)$.[1] Under these conditions,[+] microwave transport properties are complex and contain potentially more information concerning detailed scattering mechanisms than the analogous dc properties.

Because of the relatively large conductivity of semiconductors and also because of practical geometrical considerations, microwave transport experiments are generally not well suited to exact analysis. This paper discusses perturbation techniques which are useful in determining transport properties of a bulk semiconductor contained in a waveguide from measurements of the properties of the transmitted wave.

A. High Frequency Transport Phenomena

The conductivity of an isotropic semiconductor that is uniformly magnetized in the z-direction is a tensor of the form

$$\overleftrightarrow{\sigma} = \begin{Vmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{xx} \end{Vmatrix} \quad (1)$$

to the first order in B_z. A microwave transport experiment measures the relative permittivity tensor

$$\overleftrightarrow{\varepsilon}_r = \overleftrightarrow{\varepsilon}_\ell + \frac{\overleftrightarrow{\sigma}}{j\omega\varepsilon_o} \quad (2)$$

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[+]For example, the frequency of acoustical mode phonon scattering for both n-type silicon and p-type germanium is about 22 Gc/s at 77^o K.

where $\overleftrightarrow{\epsilon}_\ell$ is a diagonal tensor describing the lattice contribution.

For a simple spherical band semiconductor, the conductivity and Hall terms, σ_{xx} and σ_{xy}, are given by

$$\sigma_{xx}(\omega) = \sigma_o \left\{ \left\langle \frac{\tau}{1 + j\omega\tau} \right\rangle \Big/ \langle \tau \rangle \right\}$$

and

$$\sigma_{xy}(\omega) = \sigma_o \, \mu_{Ho} \, B_z \left\{ \left\langle \frac{\tau^2}{(1 + j\omega\tau)^2} \right\rangle \Big/ \langle \tau^2 \rangle \right\} \qquad (3)$$

with σ_o and μ_{Ho} the dc conductivity and dc Hall mobility, respectively. Normalized plots of σ_{xx} and σ_{xy} for a very simple scattering model are shown in Figures 1 and 2. More sophisticated models lead to different shaped curves; however, all models are characterized by having a large imaginary contribution in the vicinity of $\omega \langle \tau \rangle = 1$. DC experiments measure only the asymptotic values of these curves.

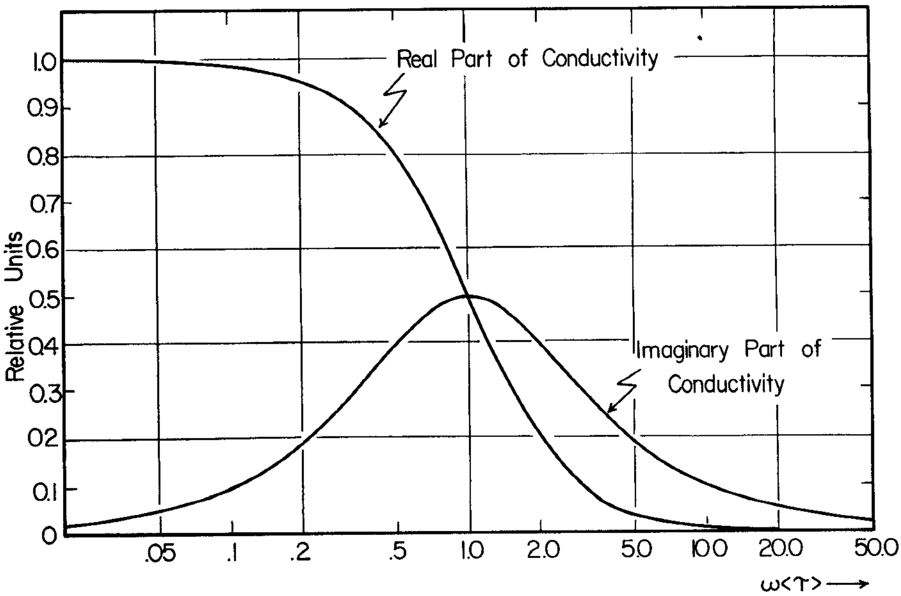

Fig. 1. Frequency dependence of normalized complex conductivity for spherical band semiconductor. Relaxation time is assumed independent of electron energy.

B. The Perturbation Method

Figure 3 shows TE waves propagating through a waveguide containing a semiconductor of arbitrary (but uniform) cross section. If mode solutions in the two regions are sufficiently alike, higher order terms can be neglected and the transmission coefficient of the principal mode written [2]

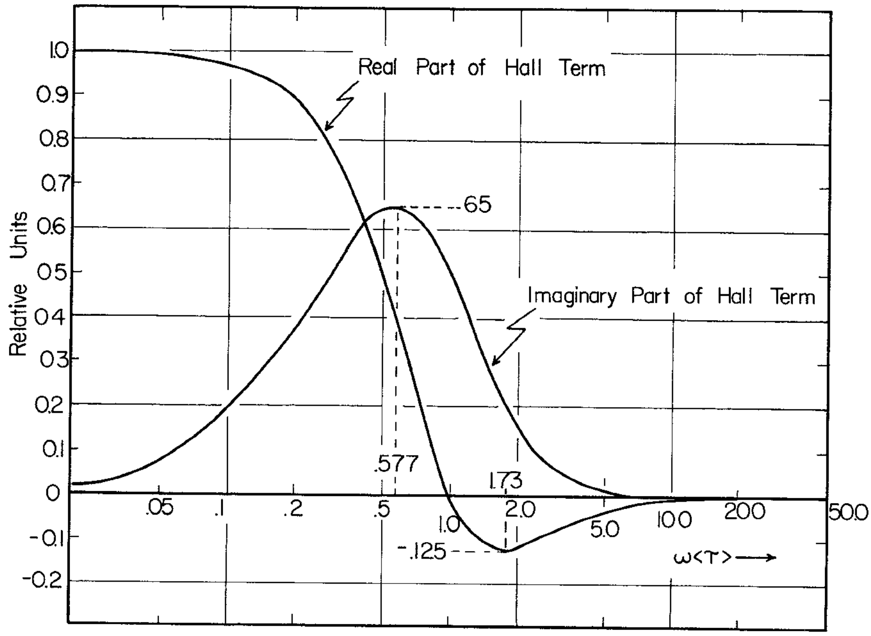

Fig. 2. Frequency dependence of normalized complex Hall term for same model. Note region of negative real part.

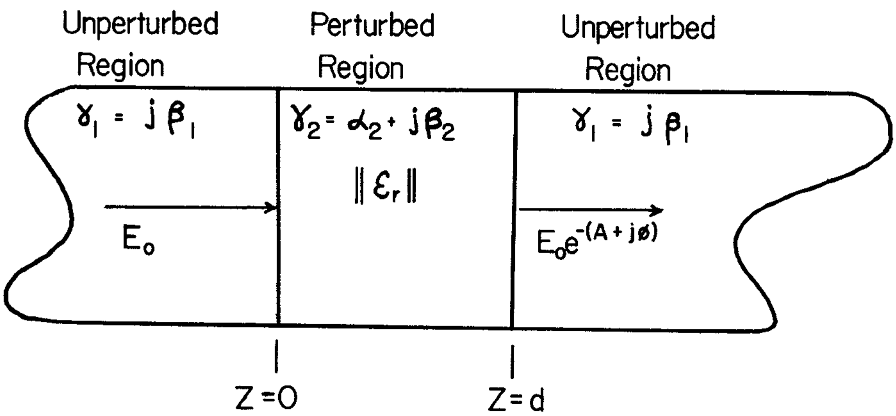

Fig. 3. Waveguide perturbed with semiconductor of uniform cross section between Z = 0 and Z = d.

$$e^{-(A + j\phi)} = \frac{4\gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^2 e^{\gamma 2d} - (\gamma_2 - \gamma_1)^2 e^{-\gamma 2d}} \quad (4)$$

where $\gamma_2 = a_2 + j\beta_2$ and $\gamma_1 = j\beta_1$ are the propagation constants in the perturbed and unperturbed regions, respectively.

In an experiment in which a perturbed section is substituted for an unperturbed section, the logarithmic magnitude and phase of the transmitted wave will change by the amounts A and $\{\phi - \beta_1 d\}$, respectively. One can then employ a digital computer inversion of (4) to obtain $\gamma_2 d$ and $\beta_2 d$ from the measured A and ϕ and the calculated value of $\beta_1 d$.

If the field distributions of a perturbed mode $\underline{E}_2(x,y)$, $\underline{H}_2(x,y)$, and the corresponding unperturbed mode $\underline{E}_1(x,y)$, $\underline{H}_1(x,y)$ are known, the difference between their propagation constants can be calculated exactly from[3]:

$$(\gamma_2 - \gamma_1) = \frac{j\omega\varepsilon_o \int \left((\overleftrightarrow{\varepsilon_r} - 1) \cdot \underline{E}_2 \right) \cdot \underline{E}_1^* \, dS}{\int (\underline{E}_1^* \times \underline{H}_2) \cdot \hat{k} \, dS + \int (\underline{E}_2 \times \underline{H}_1^*) \cdot \hat{k} \, dS} \qquad (5)$$

by integrating over the cross section of the waveguide. Under the same conditions that (4) applies, perturbed fields are known approximately and (5) can be evaluated to yield the components of $\overleftrightarrow{\varepsilon}_r$ explicitly in terms of α_2, β_2 and β_1. This calculation assumes nothing about the relative magnitudes of conduction and displacement currents and applies to both high and low conductivity materials.

C. Zero Magnetic Field--Measurement of Complex Conductivity

When $B_Z = 0$, the relative permittivity is scalar

$$\overleftrightarrow{\varepsilon}_r = \varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \qquad (6)$$

with $\varepsilon_r' = \varepsilon_\ell$ and $\varepsilon_r'' = \sigma_o/\omega\varepsilon_o$ at frequencies much less than the scattering frequency.

If the sample completely fills the waveguide, perturbed and unperturbed normal modes are identical so that (4) is exact. Equation (5) can also be evaluated exactly. The result is:

$$\left\{\varepsilon_r' - 1\right\} = \left\{\frac{1}{\omega^2 \mu_o \varepsilon_o}\right\} \left\{\beta_2^2 - \alpha_2^2 - \beta_1^2\right\}$$

$$\varepsilon_r'' = \left\{\frac{2}{\omega^2 \mu_o \varepsilon_o}\right\} \left\{\alpha_2 \beta_2\right\} \qquad (7)$$

showing that knowledge of both α_2 and β_2 is required in general to find either ε_r' or ε_r''.

As an example of an approximate solution, consider a thin sample in the center of a TE_{10} mode waveguide (Figure 4). One can show for this case that the unperturbed fields are a first order approximation to the perturbed fields under the following condition [4]:

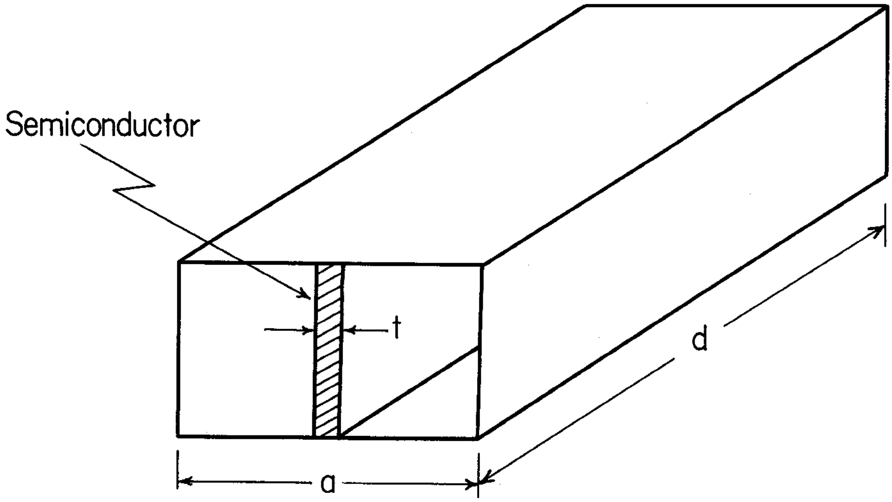

Fig. 4. Thin semiconductor placed vertically in center of rectangular TE_{10} mode waveguide.

$$t << \frac{2a}{\pi}$$

$$t\left|\varepsilon_r - 1\right| << \left(\frac{\pi}{a}\right)\left(\frac{2}{\omega^2\mu_o\varepsilon_o}\right) . \qquad (8)$$

When these inequalities are satisfied, (4) applies and (5) can again be evaluated to yield

$$\left\{\varepsilon_r' - 1\right\} = \left\{\frac{a}{2t}\right\}\left\{\frac{1}{\omega^2\mu_o\varepsilon_o}\right\}\left\{\beta_2^2 - \alpha_2^2 - \beta_1^2\right\}$$

$$\varepsilon_r'' = \left\{\frac{a}{2t}\right\}\left\{\frac{2}{\omega^2\mu_o\varepsilon_o}\right\}\left\{\alpha_2\beta_2\right\} . \qquad (9)$$

D. Gyromagnetic Semiconductor--Measurement of Complex Hall Effect

An isotropic semiconductor that is magnetized in the z-direction has a tensor relative permittivity

$$\overleftrightarrow{\varepsilon}_r = \begin{Vmatrix} (\varepsilon_r' - j\varepsilon_r'') & -j(\eta' - j\eta'') & 0 \\ j(\eta' - j\eta'') & (\varepsilon_r' - j\varepsilon_r'') & 0 \\ 0 & 0 & (\varepsilon_r' - j\varepsilon_r'') \end{Vmatrix} \qquad (10)$$

to the first order in the magnetic field. At low frequency, the four components are

$$\varepsilon_r' = \varepsilon_\ell \qquad \varepsilon_r'' = (\sigma_o/\omega\varepsilon_o)$$
$$\eta' = (\sigma_o/\omega\varepsilon_o)\,\mu_{Ho}\,B_z \qquad \eta'' = 0 \quad . \tag{11}$$

The off-diagonal terms, which are responsible for the dc Hall effect, manifest themselves in the Faraday effect at microwave frequencies[5,6]. Perturbation theory will yield these terms as well as the diagonal terms.

Consider linearly polarized radiation incident upon the sample completely filling a section of degenerate (e.g., circular) waveguide. One can measure four properties of the elliptically polarized transmitted wave:

A - the logarithmic amplitude

ϕ - the phase

X - the ellipticity (ratio of E_{max} to E_{min} on the waveguide axis)

Θ - the polarization angle.

As long as:

$$\left| \eta' - j\eta'' \right| \ll \left| \varepsilon_r' - j\varepsilon_r'' \right| \tag{12}$$

the perturbed and unperturbed modes will be sufficiently alike that (4) will apply to <u>both</u> circularly polarized components of the transmitted wave. Thus, $\alpha_2 d$ and $\beta_2 d$ can again be determined from measurements of A, ϕ, and $\beta_1 d$ by a computer inversion of (4). Furthermore, the ellipticity per unit length ξ_2 and rotation angle per unit length θ_2 that characterize an infinite medium can be determined from the corresponding experimental quantities X and Θ as follows:

$$\xi_2 d = \left(\frac{\partial \alpha_2 d}{\partial A}\right) X - \left(\frac{\partial \alpha_2 d}{\partial \phi}\right) \Theta$$
$$\theta_2 d = \left(\frac{\partial \beta_2 d}{\partial A}\right) X + \left(\frac{\partial \beta_2 d}{\partial \phi}\right) \Theta \tag{13}$$

with the partial derivatives in (13) also calculated by the computer. We have programmed a computer to find $\alpha_2 d$ and $\beta_2 d$ and the four partial derivatives of (13) to within 0.1 per cent when given measured values of A, ϕ, and $\beta_1 d$. The computation time averages about five seconds.

Under the same conditions that (4) and (13) apply, (5) can be evaluated to determine the tensor components explicitly. The result is:

$$\left\{\varepsilon_r' - 1\right\} = \left\{\frac{1}{\omega^2 \mu_o \varepsilon_o}\right\} \left\{\beta_2^2 - \alpha_2^2 - \beta_1^2\right\}$$

$$\varepsilon_r'' = \left\{\frac{2}{\omega^2 \mu_o \varepsilon_o}\right\} \left\{\alpha_2 \beta_2\right\}$$

$$\eta' = \left\{\frac{2}{\omega^2 \mu_o \varepsilon_o K}\right\} \left\{\beta_2 \theta_2 - \alpha_2 \xi_2\right\} \qquad (14)$$

$$\eta'' = \left\{\frac{2}{\omega^2 \mu_o \varepsilon_o K}\right\} \left\{\alpha_2 \theta_2 - \beta_2 \xi_2\right\}$$

where K is a waveguide constant given by $8/\pi^2$ for the square TE_{10} mode and 0.838 for the circular TE_{11} mode. The assumption of (12) limits the magnetic field and excludes such high field behavior as the cyclotron resonance effect.

At frequencies much less than the scattering frequency, the four measurements are not independent because $\eta'' = 0$. This leads to two equations for the dc Hall mobility.

$$\mu_{Ho} = \frac{1}{K B_z} \left\{\frac{\beta_2}{\alpha_2} + \frac{\alpha_2}{\beta_2}\right\} \left\{\frac{\xi_2}{\alpha_2}\right\}$$

and

$$\mu_{Ho} = \frac{1}{K B_z} \left\{\frac{\beta_2}{\alpha_2} + \frac{\alpha_2}{\beta_2}\right\} \left\{\frac{\theta_2}{\beta_2}\right\} . \qquad (15)$$

Thus, the self-consistency of the measurements for $\omega \langle \tau \rangle << 1$ can be used as an experimental check.

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